



Newton's *Notes*

FE Mechanical Review Guide

Every topic on the NCEES FE Mechanical exam in 19 focused chapters: the ideas, the equations, the Handbook search terms and the traps, with 99 worked examples and 297 practice problems, all fully solved.

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How to use this guide

This book is the teaching half of Newton's Notes. It follows the Newton's Notes course chapter by chapter, so you can read it alongside the lessons or on its own. The companion *Practice Problems* book does the opposite: it mixes every subject together the way the exam does. Learn a topic here, then prove you can recognize it there.

What is in each section

- **Handbook search terms.** The words to type into the search box of the FE Reference Handbook to reach the right page fast. On exam day, finding the equation quickly matters as much as knowing it.
- **The ideas.** A short explanation of what the topic is and how the exam uses it.
- **Key equations.** The working equations, each with a note on units or when it applies.
- **Traps.** The mistakes that wrong answer choices are built from.
- **Exam tip.** One habit that saves time or points.
- **Worked example.** A new exam-style problem solved step by step, with the reason behind each wrong choice.
- **Practice.** Three new problems, easiest first. Solutions are at the end of each chapter.

Each chapter ends with a checklist. Tick a line only when you can do it without looking. Anything left unticked is your next study session.

Difficulty marks

Solutions are tagged ●○○ for a one-step problem, ●●○ for a standard problem and ●●● for a multi-step problem. The practice problems in each section usually run one of each, so you build up rather than start at the hardest.

Chapter 19

A handful of topics in the NCEES specification fall outside the course chapters: pseudocode, expected value, mechanisms, compressible flow, power screws, reliability, fluid power and drawing interpretation. Chapter 19 teaches all of them, so nothing on the specification is left out of this guide. The appendix maps every NCEES subtopic to the sections that teach it.

Units and conventions

Problems use SI units with $g = 9.81 \text{ m/s}^2$ and water at $1,000 \text{ kg/m}^3$ unless stated. Answers are rounded to three significant figures. If your result is close to a choice but not exact, pick the closest one, just as you would on the exam.

The exam and this guide

The FE Mechanical exam is a computer-based test of 110 multiple-choice questions in a six-hour appointment, with the searchable FE Reference Handbook on screen. NCEES publishes a range of questions for each of 14 knowledge areas. The table shows those ranges and the chapters of this guide that teach each area.

KNOWLEDGE AREA	QUESTIONS	RANGE ON A 0–15 SCALE	CHAPTERS
1. Mathematics	6–9		1, 2, 3, 4, 5, 19
2. Probability and Statistics	4–6		6, 19
3. Ethics and Professional Practice	4–6		7
4. Engineering Economics	4–6		8
5. Electricity and Magnetism	5–8		9
6. Statics	9–14		10
7. Dynamics, Kinematics and Vibrations	10–15		11, 19
8. Mechanics of Materials	9–14		12
9. Material Properties and Processing	7–11		13
10. Fluid Mechanics	10–15		14, 19
11. Thermodynamics	10–15		15
12. Heat Transfer	7–11		16
13. Measurements, Instrumentation and Controls	5–8		17
14. Mechanical Design and Analysis	10–15		12, 18, 19
Total	110		

Source: NCEES FE Mechanical CBT exam specifications, effective July 2020. Confirm current specifications at [ncees.org](https://www.ncees.org).

Study in proportion to the weights

Dynamics, fluid mechanics, thermodynamics and mechanical design can each supply 10 to 15 questions, and statics and mechanics of materials 9 to 14 each. Those six areas make up more than half the exam, so they deserve more than half your study time. Mathematics carries 6 to 9 questions on its own and also underpins every other area. Ethics, economics and probability are small, but they are also the fastest points on the exam and worth securing early.

A practical plan is one chapter every two or three days, working the examples and practice problems as you go, then the mixed sets in the *Practice Problems* book in the final weeks to find and fix your weakest areas.

1

Analytic Geometry and Trigonometry

PART 1 • MATHEMATICS

Analytic geometry and trigonometry are tested directly in the mathematics questions and indirectly almost everywhere else: force components, linkage angles, flow areas and tank volumes all rest on them. The goal of this chapter is speed, knowing which Handbook entry to open and which calculator mode to use before you start.

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NCEES TOPICS IN THIS CHAPTER

Mathematics: Analytic geometry

1.1 Straight Lines

HANDBOOK SEARCH

Straight Line

slope

point-slope

A straight line is fixed by its slope and one point on it. The slope between two points is rise over run, $m = (y_2 - y_1)/(x_2 - x_1)$. Slope-intercept form, $y = mx + b$, shows the slope and the y -intercept at a glance. Point-slope form builds a line from any point and slope. The general form $Ax + By + C = 0$ has slope $-A/B$. The angle of inclination from the positive x -axis is $\theta = \arctan m$, reported between 0° and 180° , so a negative slope gives an angle above 90° .

Parallel lines share a slope. Perpendicular lines have slopes whose product is -1 , so each slope is the negative reciprocal of the other. The angle between two lines follows from their slopes alone. The perpendicular distance from a point to a line uses the general form: substitute the point into $Ax + By + C$, take the absolute value and divide by $\sqrt{A^2 + B^2}$.

KEY EQUATIONS

$m = (y_2 - y_1)/(x_2 - x_1)$	Slope from two points; keep the same point first
$y = mx + b, y - y_1 = m(x - x_1)$	Slope-intercept and point-slope forms
$Ax + By + C = 0, m = -A/B$	General form; the y -intercept is $-C/B$
$m_1 m_2 = -1$	Perpendicular lines; parallel lines have $m_1 = m_2$
$\tan \alpha = (m_2 - m_1)/(1 + m_1 m_2)$	Angle between two lines from their slopes
$d = Ax_0 + By_0 + C / \sqrt{A^2 + B^2}$	Perpendicular distance from (x_0, y_0) to the line

TRAPS

- Subtracting the y values in one order and the x values in the other, which flips the sign of the slope.
- Reading the slope of $Ax + By + C = 0$ as A/B ; solving for y gives $-A/B$.
- Taking the reciprocal for a perpendicular slope but forgetting to change its sign.
- Reporting $\arctan$ of a negative slope as a negative angle when the question wants 0° to 180° ; add 180° .

EXAM TIP Convert any line to $y = mx + b$ before reading its slope, intercept or angle. For point-to-line distance, use the general-form formula; it beats finding the foot of the perpendicular.

WORKED EXAMPLE

What is the perpendicular distance from the point $(2, 7)$ to the line $5x + 12y - 26 = 0$?

- (A) 7.23 (B) 68.0 (C) 0.402 (D) 5.23

1. $d = |Ax_0 + By_0 + C| / \sqrt{A^2 + B^2}$
2. $= |5(2) + 12(7) + (-26)| / \sqrt{25 + 144} = 68/13 = 5.23$.

Answer (D) 5.23

WHY NOT THE OTHERS (A) drops the constant C . (B) forgets to divide by $\sqrt{A^2 + B^2}$. (C) divides by $A^2 + B^2$ without the square root.

PRACTICE

- 1.1-1** A line passes through $(-3, -1)$ and $(1, -9)$. What is its y -intercept?
 (A) 5 (B) -1.5 (C) -3.5 (D) -7
-
- 1.1-2** What angle does the line $x - y = 12$ make with the positive x -axis (0° to 180°)?
 (A) 22.5° (B) 45.0° (C) 90.0° (D) 135°
-
- 1.1-3** A line passes through $(3, 2)$ perpendicular to a line of slope -3 . What is its y -value at $x = 6$?
 (A) -7 (B) 3 (C) 1 (D) 11
-

1.2 Quadratic Equations

HANDBOOK SEARCH

Quadratic Equation

roots

Any quadratic $ax^2 + bx + c = 0$ can be solved with the quadratic formula. Factor only when the roots are obvious, and divide out a common factor first to keep the numbers small. The discriminant $b^2 - 4ac$ predicts the roots before you finish: positive gives two real roots, zero gives one repeated root, and negative gives a complex-conjugate pair. The sum of the roots is $-b/a$ and their product is c/a , a fast check on any answer.

The graph of $y = ax^2 + bx + c$ is a parabola with its vertex at $x = -b/(2a)$. The vertex is a minimum when $a > 0$ and a maximum when $a < 0$. Physical problems such as projectile height often produce one root that makes no sense, like a negative time. Keep the root the situation allows.

KEY EQUATIONS

$$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$$

 Roots of $ax^2 + bx + c = 0$

$$D = b^2 - 4ac$$

Positive: two real roots; zero: repeated root; negative: complex pair

$$x = -b/(2a) \pm i\sqrt{(4ac - b^2)/(2a)}$$

Complex roots when the discriminant is negative

$$x_1 + x_2 = -b/a, x_1x_2 = c/a$$

Sum and product of the roots; a quick check

$$x_v = -b/(2a), y_v = c - b^2/(4a)$$

 Vertex: minimum if $a > 0$, maximum if $a < 0$

TRAPS

- Dividing only the square-root term by $2a$; the whole numerator $-b \pm \sqrt{(b^2 - 4ac)}$ goes over $2a$.
- Losing a sign when c is negative: with $a = 2$ and $c = -5$, the term $-4ac$ is $+40$.
- Keeping a negative time or length that the physical situation rules out.
- Calling the vertex a minimum without checking the sign of a .

EXAM TIP If your calculator has an equation or polynomial mode, solve quadratics there and spend the saved time deciding which root the question wants.

WORKED EXAMPLE

What is the larger root of $3x^2 + 9x - 12 = 0$?

- (A) 1 (B) -4 (C) -1 (D) 3

1. Divide by 3: $x^2 + 3x - 4 = 0$, which factors as $(x + 4)(x - 1) = 0$.

2. Roots: -4 and 1. The larger is 1.

Answer (A) 1

WHY NOT THE OTHERS (B) is the smaller root. (C) flips the sign of the root. (D) forgets to divide by a.

PRACTICE

1.2-1 How many and what kind of roots does $x^2 + 2x + 5 = 0$ have?

- (A) One repeated real root (B) Two distinct real roots
(C) No roots at all (D) Two complex conjugate roots

1.2-2 What is the minimum value of $y = 2x^2 + 8x + 7$?

- (A) 7.00 (B) -2.00 (C) -1.00 (D) 31.0

1.2-3 A ball's height is $h = 30 + 20t - 4.905t^2$ m. When does it hit the ground?

- (A) 5.24 s (B) 1.17 s (C) 4.08 s (D) 2.47 s

1.3 Conic Sections

HANDBOOK SEARCH

Conic Sections

Circle

Parabola

Ellipse

Hyperbola

Circles, ellipses, parabolas and hyperbolas are all cases of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The quantity $B^2 - 4AC$ identifies the curve with no further algebra: negative for an ellipse, zero for a parabola and positive for a hyperbola. A circle is the ellipse with $A = C$ and $B = 0$. Eccentricity tells the same story: $e = 0$ for a circle, between 0 and 1 for an ellipse, 1 for a parabola and greater than 1 for a hyperbola.

To find a center, radius, vertex or focus, complete the square in x and in y , then match a standard form. An ellipse's foci lie $c = \sqrt{a^2 - b^2}$ from the center along the major axis; a hyperbola's lie $c = \sqrt{a^2 + b^2}$ away. The Handbook writes the parabola as $(y - k)^2 = 2p(x - h)$, so its focus sits $p/2$ from the vertex. Many textbooks use $4p$ instead, with the focus p from the vertex.

KEY EQUATIONS

$$B^2 - 4AC$$

Negative: ellipse; zero: parabola; positive: hyperbola

$$(x - h)^2 + (y - k)^2 = r^2$$

Circle with center (h, k) and radius r

$$(y - k)^2 = 2p(x - h)$$

Handbook parabola; vertex (h, k) , focus $(h + p/2, k)$

$$(x - h)^2/a^2 + (y - k)^2/b^2 = 1, c^2 = a^2 - b^2$$

Ellipse with $a > b$; foci at $(h \pm c, k)$

$$(x - h)^2/a^2 - (y - k)^2/b^2 = 1, c^2 = a^2 + b^2$$

Hyperbola; foci at $(h \pm c, k)$

Chapter 1 solutions

Answers to the practice problems in Sections 1.1–1.8. Read the trap note for every problem you missed or guessed.

1.1-1 SECTION 1.1 • •••

Answer (D)

Slope $m = (-9 - (-1))/(1 - (-3)) = -2$. $b = y_1 - mx_1 = -1 - (-2)(-3) = -7$.

TRAP (A) adds mx_1 instead of subtracting it. (C) is the x-intercept, where $y = 0$.

1.1-2 SECTION 1.1 • •••

Answer (B)

Slope $m = -A/B = 1$. $\theta = \arctan(1) = 45.0^\circ$.

TRAP (D) uses B/A .

1.1-3 SECTION 1.1 • •••

Answer (B)

Perpendicular slope $= -1/(-3) = 0.333$. $y = 2 + 0.333(6 - 3) = 3$.

TRAP (A) uses the original slope (parallel line). (C) uses the reciprocal without the sign change. (D) negates the slope without inverting it.

1.2-1 SECTION 1.2 • •••

Answer (D)

Discriminant $b^2 - 4ac = 4 - 20 = -16$. Negative: two complex conjugate roots.

1.2-2 SECTION 1.2 • •••

Answer (C)

The vertex is at $x = -b/(2a) = -8/4 = -2$. $y(-2) = -1.00$.

TRAP (A) is the y-intercept ($x = 0$), not the vertex. (B) is the x-location of the vertex, not the value of y . (D) uses $x = +b/(2a)$, the wrong sign.

1.2-3 SECTION 1.2 • •••

Answer (A)

Set $h = 0$: $4.905t^2 - 20t - 30 = 0$. $t = [20 + \sqrt{(20)^2 + 4(4.905)(30)}]/(2 \times 4.905) = 5.24 \text{ s}$ (the negative root is discarded).

TRAP (B) reports the magnitude of the negative root. (C) is the time to return to the launch height. (D) ignores the upward launch speed.

1.3-1 SECTION 1.3 • •••

Answer (C)

Complete the squares: $(x - 3)^2 + (y + 1)^2 = 6 + 9 + 1 = 16$. $r = \sqrt{16} = 4.00$.

TRAP (A) takes $\sqrt{|F|}$ without completing the squares. (B) forgets the square root.

1.3-2 SECTION 1.3 • •••

Answer (A)

$B^2 - 4AC = 4 - 4 = 0$. Zero $\rightarrow$ parabola.

1.3-3 SECTION 1.3 • •••

Answer (D)

$a = 5$, $b = 3$, so $c = \sqrt{a^2 - b^2} = \sqrt{(25 - 9)} = 4$. The foci sit at $\pm c$, so they are $2c = 8.00$ apart.

TRAP (A) adds $a^2 + b^2$, the hyperbola relation. (B) gives the distance from the center to one focus, not between the foci. (C) is the length of the major axis.

1.4-1 SECTION 1.4 • •••

Answer (C)

$\Delta x = -8$, $\Delta y = 4$, $\Delta z = -4$. $d = \sqrt{(64 + 16 + 16)} = 9.80$.

TRAP (A) adds the component differences. (B) ignores the z-coordinates. (D) forgets the square root.

1.4-2 SECTION 1.4 • •••

Answer (A)

Average the coordinates: $z = (-2 + 4)/2 = 1$.

TRAP (B) takes half the difference. (C) forgets to halve. (D) takes the difference.

1.4-3 SECTION 1.4 • •••

Answer (C)

Complete the squares: center $(1, -2, 3)$. $r^2 = 1 + 4 + 9 - (-2) = 16 \rightarrow r = 4$.

TRAP (A) takes the square root of the constant only. (B) forgets the square root. (D) is the distance from the origin to the center.

1.5-1 SECTION 1.5 • •••

Answer (C)

$\tan \theta = \text{rise/run} = 0.600/6 = 0.100$. $\theta = \arctan(0.100) = 5.71^\circ$.

TRAP (A) is the angle the ramp makes with the vertical.

1.5-2 SECTION 1.5 • •••

Answer (D)

The wire is the hypotenuse: $\sin 55^\circ = 20/L$. $L = 20/\sin 55^\circ = 24.4 \text{ m}$.

TRAP (A) multiplies instead of dividing. (B) uses cosine. (C) is the horizontal distance to the anchor.

1.5-3 SECTION 1.5 • •••

Answer (D)

$\omega = 300(2\pi/60) = 31.4 \text{ rad/s}$. $v = \omega r = 31.4(0.3) = 9.42 \text{ m/s}$.

TRAP (A) forgets the 2π . (B) uses the diameter as the radius. (C) leaves rpm unconverted.

1.6-1 SECTION 1.6 • •••

Answer (C)

Use the basic identities: $\sec^2 \theta - \tan^2 \theta = 1$.



That's the end of the sample.

You've seen the start of Chapter 1 and its solutions. The full guide keeps going for 18 more chapters in the same format.

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